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perturbations has deduced the mean elements of the planet. To me it is as good as certain that the perturbations thus calculated (the hundredths of seconds are omitted) are sufficiently accurate to be employed next year for a more exact determination of the mean elements. The terms of the first power of the masses which are here given will therefore subserve the object in view, and on a future occasion the question may be investigated, whether the terms of the second order exercise any sensible influence.

"The perturbational values are all given as they follow from the calculations. Moreover, the secular equations of the elements which might be deduced from the terms involving *t* (the unit of *t* is the Julian year) are not deduced, since for the next year the calculation in its present form is more convenient. By the way, I see that I have not made any mention of these terms in speaking of the integration of the differential equations. However, what I have said above will be quite sufficient to give you an idea of this method of computing the perturbations.

"*Berlin, April* 24, 1853."

The following are the results obtained by Dr. Brünnow relative to the planet *Flora*, to which Professor Encke alludes at the beginning of the foregoing extract:—

Mean Elements of Flora with the Perturbational Equations for Jupiter and Saturn. Calculated by Dr. Brünnow, Assistant at the Berlin Observatory, April 1853.

For the five normal places which were employed in the special perturbations, by means of the resulting perturbational equations for *Jupiter* and *Saturn*, purely elliptic mean elements were found. In this manner were obtained:—

Epoch 1848, Jan. 1·0, Mean Berlin Time.

	°	′	″	
Mean Anomaly	35	54	3·62	
Longitude of the Perihelion	32	54	28·34	Mean Equinox of the Epoch.
Long. Asc. Node	110	17	48·62	
Inclination	5	53	7·96	
Angle of Excentricity	9	0	56·29	
Mean, Dail. Sid. Mo.	1086″·330982			
Log. Sem. Maj.-axis	0·3426963			

These elements gave the mean place, and by applying the perturbational equations to each of the five normal places, the latter were thus represented:—

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		Theor. — Obs. Δα	Theor. — Obs. Δδ
		″	″
1848	Jan. 1·0	−0·1	+1·2
	Apr. 30·0	−0·2	−0·6
1849	May 5·5	+0·3	+1·5
1850	Sep. 18·5	−0·9	+1·9
1852	Mar. 29·0	+0·9	+1·6

The opposition for 1853 was calculated from the special perturbations. Now, if we calculate it from the purely elliptic elements, taking into account the perturbations, we have:—

	α	δ	
	° ′ ″	° ′ ″	
1853 July 11·5	311 49 42·0	−19 47 35·8	By Special Perturbations.
	45·3	32·1	By General ——

The perturbations of $r^0 \delta r$ are given in seconds. They will require to be changed into units of decimals. z is the co-ordinate of the disturbed planet drawn perpendicular to the plane of the mean orbit. It is given in seconds.

δv is the correction of the true anomaly.

Perturbations of Flora by Jupiter.

	$r\delta r$.		δv.	
	cos	sin	cos	sin
	+47″·3 − 0″·3407 t		−36″·700 t	
M	−12·0 +1·438 t	−46·577 t	−0″·3 −19″·218 t	+3·2 −0·592 t
2M	− 1·1 +0·112 t	− 3·627 t	+0 ·0 − 3 ·738 t	+1·3 −0·115 t
3M	0·0 +0·013 t	− 0·423 t	0 ·0 − 0 ·756 t	+0·4 −0·023 t
4M	0·0 +0·002 t	− 0·058 t	0 ·0 − 0 ·155 t	0·0 −0·005 t

	z.	
	cos	sin
	−1″·7 + 0″·202 t	
M	+1″·5 +0″·852 t	−0″·2 +6″·038 t
2M	+0 ·3 +0 ·066 t	0 ·0 +0 ·470 t
3M	0 ·0	0 ·0 +0 ·055 t

Perturbations of Flora by Jupiter—(continued).

	$r\,\delta r.$		$\delta v.$		$z.$	
	cos	sin	cos	sin	cos	sin
	″	″	″	″	″	″
−M−M′	− 3·7	+ 4·3	− 2·1	− 2·2	+1·7	−0·7
−M′	− 49·5	+ 22·9	− 11·3	− 16·6	+4·3	−1·2
M−M′	+158·4	− 66·4	− 35·4	− 85·0	−4·4	+2·0
2M−M′	+ 23·6	− 10·4	− 7·3	− 17·3	−1·7	+0·9
3M−M′	+ 2·5	− 1·1	− 1·5	− 3·5		
−M−2M′	+ 3·6	− 3·8	+ 2·7	+ 2·6		
−2M′	+ 54·5	− 54·7	+ 14·0	+ 13·3	−3·2	+2·6
M−2M′	−100·6	+101·9	+108·2	+110·2	+6·2	−4·1
2M−2M′	−214·7	+198·8	+ 75·4	+ 80·8	+1·5	−1·3
3M−2M′	− 16·4	+ 14·8	+ 15·8	+ 17·0	−0·1	+0·1
4M−2M′	− 1·5	+ 1·3	+ 3·2	+ 3·5		
−M−3M′	+ 1·1	+ 0·2	0·0	+ 1·0		
−3M′	+ 17·9	+ 2·6	− 0·9	+ 5·9	−0·2	−0·6
M−3M′	− 21·2	+ 20·8	− 6·9	+ 42·2	+1·1	−0·8
2M−3M′	− 17·5	−112·4	− 39·8	+ 9·6	−4·3	+6·7
3M−3M′	− 15·7	+ 20·6	− 0·3	+ 5·7	−0·3	+0·3
4M−3M′	− 1·7	+ 1·8	+ 0·3	+ 1·4		
−4M′	− 1·3	+ 0·7	− 0·3	− 0·6		
M−4M′	+ 3·1	+ 3·2	− 2·0	− 4·6	−0·2	−0·4
2M−4M′	− 15·5	− 10·1	− 5·1	+ 7·3	+1·0	+1·6
3M−4M′	− 4·9	− 9·4	− 3·0	+ 2·6	0·0	+0·6
4M−4M′	− 1·1	+ 5·9	+ 1·0	+ 0·8		
5M−4M′	− 0·2	+ 0·9	+ 0·4	0·0		
3M−5M′	+ 1·8	0·0	0·0	− 0·5		
4M−5M′	− 1·2	− 1·5	− 0·4	+ 0·2		
5M−5M′	+ 0·3	+ 1·4	+ 0·3	0·0		

M denotes the mean anomaly of *Flora*, and M′ the mean anomaly of *Jupiter*.

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Perturbations of Flora by Saturn.

	$r^0 \delta r$.		δv.	
	cos	sin	cos	sin
	+ 1″·96 + 0″·0181 *t*		1″·520 *t*	
M	−0″·5 −0″·038 *t*	0″·0 −0″·977 *t*	0″·0 −0″·403 *t*	−0″·0 + 0″·013 *t*
2 M	0 ·0 −0 ·003 *t*	0 ·0 −0 ·076 *t*	0 ·0 −0 ·078 *t*	0 ·0 + 0 ·003 *t*
3 M	……	0 ·0 −0 ·009 *t*	0 ·0 −0 ·016 *t*	……

	z.	
	cos	sin
	+ 0″·028 *t*	
M	0″·0 −0″·116 *t*	0″·0 −0″·106 *t*
2 M	0 ·0 −0 ·009 *t*	0 ·0 −0 ·008 *t*

	$r^0 \delta r$.		δv.		z.	
	cos	sin	cos	sin	cos	sin
	″	″	″	″	″	″
−M−M′,	+0·1	− 0·3				
M′,	−0·7	− 2·1	+1·3	+0·3		
M−M′,	+3·3	+ 7·8	+3·6	−1·5	0·0	+0·1
2 M−M′,	+0·3	+ 0·8	+0·7	−0·3		
3 M−M′,	……	……	+0·1	0·0		
−2 M′,	−1·5	+ 3·0	−0·4	−0·1	+0·1	0·0
M−2 M′,	+5·4	−10·0	−5·3	−3·0	+0·4	−0·1
2 M−2 M′,	+2·7	− 5·8	−2·4	−1·2		
3 M−2 M′,	+0·2	− 0·4	−0·5	−0·3		
4 M−2 M′,	……	……	−0·1	0·0		
−3 M′,	0·0	+ 0·5	−0·1	0·0		
M−3 M′,	+0·2	− 1·5	−1·0	−0·1		
2 M−3 M′,	−0·4	− 1·2	−0·5	+0·1		
3 M−3 M′,	+0·4	− 0·1	−0·1	−0·1		

M denotes the mean anomaly of *Flora*, and M′, the mean anomaly of *Saturn*. The unit of time is the Julian year of 365¼ days.